

Cosmological bootstrap

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A huge value of cosmological constant characteristic for the particle physics and the inflation of early Universe are inherently related to each other: one can construct a fine-tuned superpotential, which produces a flat potential of inflaton with a constant density of energy $V = \Lambda^4$ after taking into account for leading effects due to the supergravity, so that an introduction of small quantum loop-corrections to parameters of this superpotential naturally results in the dynamical instability relaxing the primary cosmological constant by means of inflationary regime. The model phenomenologically agrees with observational data on the large scale structure of Universe at $\Lambda \sim 10^{16}$ GeV.

PACS numbers: 98.80.-k, 11.30.Pb, 04.65.+e, 11.10.Gh

I. INTRODUCTION

In modern cosmology there are actual problems of cosmological constant [1] and inflation [2–6]. Namely, if we consider the first among the above mentioned issues, then in the framework of quantum field theory we could naturally expect that a scale of vacuum energy is determined by a value characteristic for interactions of elementary particles, hence, the cosmological constant would get the huge value comparable with the Planck mass or, at least, with a scale of gauge symmetry breaking in the particle physics such as a grand unification and the electroweak symmetry or, finally, with the characteristic values of quark-gluon condensates. However, in all of listened cases the cosmological constant would essentially exceed a constraint extracted from observations of the anisotropy of cosmic microwave background radiation (CMBR) [7–9], the inhomogeneity of matter distribution in the Universe (LSS – large scale structure) [10], the dependence of brightness for the type Ia supernovas on the red shift (SnIa – supernova Ia) [11–14], because, in accordance with these data, the admissible value of vacuum energy density has got the scale of 10^{-3} eV, that contradicts to the concept of characteristic energies corresponding to the field interactions. Therefore, in the literature, there are various discussions on mechanisms transforming the initial huge-valued cosmological constant to the reduced value close to that of observed.

Among such the models we emphasize the renormalization group approach with a “running” cosmological constant evolving versus the rate of Universe expansion, i.e. versus the Hubble parameter [15–23]. In this way, a minimum of effective potential, i.e. the density of vacuum energy, being invariant under the renormalization group transformations, depends on the coupling constants given by values of fields related with the Hubble rate, that leads to a slow logarithmic evolution of cosmological constant in agreement with the equations of renormalization group.

In another approach, among all of non-renormalizable theories including the Einstein gravitation, one specifies a kind of asymptotically safe theories [24] with the following properties: for a countable number of local operators of non-renormalizable theory with arbitrary coefficients one isolates dimensional factors in the form of degrees of a scale, so that overall dimensionless constants or “charges” satisfy equations of a renormalization group with *a final number of fixed points*, therefore, the theory *asymptotically* gets a predictive power in vicinity of attractive fixed points. The asymptotically safe gravitation can include the evolution of cosmological term, say, to its zero value as well as the inflation [25].

In a new approach by J.D. Barrow and D.J.Show [26–28], the cosmological constant is considered as a field, so that its value is extracted from the principle of extremal action restricted by causality. As a general consequence, one finds that the scale of cosmological constant is naturally determined by the inverse age of present Universe, in Planckian units. In the case of general relativity describing the evolution of homogeneous isotropic Universe, there is a falsifiable connection of cosmological constant to the spatial curvature satisfying the present limits observed. The curvature is intrinsically related with an amount of inflation, so that a distribution of probability for the inflation consistent with the properties of our Universe has got a narrow peak in vicinity of cosmological constant fixed by an appropriate small scale.

In [29–31] V. Emelyanov and F.R. Klinkhamer considered a mechanism compensating a primary cosmological constant due to specific vector fields, so that the Minkowsky spacetime with zero value of cosmological constant can be an attractor of dynamical equations. In other approach, the cosmological constant relaxes by gradually passing different regions of its potential, i.e. by moving from a plateau to plateau [32].

One also has considered a possibility of application for the quantum mechanical mechanism of tiny mixing of two non-stationary states (the so called seesaw) with different densities of energy as given by the particle physics. Then,

the mixing results in the stationary vacuum getting the cosmological constant of reduced scale [33–40].

Next, the inflation model, solving a lot of problems for the observational cosmology, leads to a potential of scalar inflaton, that is characterized by the mass $m \approx 1.5 \times 10^{13}$ GeV, a vacuum expectation value essentially exceeding the Planck scale of energy, a tiny value of self-interaction constant (the constant of quartic self-action for the field, $\lambda \sim 10^{-13}$) and a flat plateau at the magnitude of energy density Λ^4 at $\Lambda \sim 10^{16}$ GeV. In this respect, one has got the question on the naturalness of such the exotic potential (see exhaustive review on the inflation relation to the particle physics and a mechanism of reheating and thermalization of Universe after the inflation¹ in [41]).

To solve the problem of inflaton naturalness, at present some studies are actively focused to a model of Higgs scalar Φ_H in the Standard model with a non-minimal coupling to the gravity (to the scalar curvature R) in the form of lagrangian $L_{\text{int}} = \xi R \Phi_H^\dagger \Phi_H$, where the coupling constant has got a value about $\xi \sim 10^4$. Further, under a conformal transformation to an effective field of inflaton minimally coupled to the gravity, one gets rather a flat potential with the necessary magnitude of plateau [45–54]. In this way, an account for corrections calculated within the method of appropriate renormalization group, leads to strict constraints on the mass of Higgs boson: $135.6 \text{ GeV} < m_H < 184.5 \text{ GeV}$.

The Higgs boson minimally coupled to the gravity ($\xi \rightarrow 0$) could play an essential role in the dynamics of early Universe [55]. Indeed, there is a critical value of Higgs boson-mass, which is approximately equal to $m_H^{\text{crit.}} \approx 153 \pm 3$ GeV after an account for two-loop corrections [56], so that at supercritical values of the mass, the Higgs scalar is not able to cause the inflation of Universe. If the Higgs particle is the only scalar field in the theory up to Planckian scales of energy, then the Higgs boson of subcritical mass is forbidden, since the inflation caused by such the field would generate the Big Bang Universe with a large scale structure of matter exceptionally different from the observed one. At supercritical values of Higgs boson-mass, a distribution of matter inhomogeneity would be determined by finely tuned initial data (that would be avoided due to the inflation). Thus, the subcritical values of Higgs boson-mass will inevitably require the introduction of scalar field of inflaton additional to the Standard model, and the inflaton should dynamically provide us with the formation of necessary properties of large scale structure in our Universe [57].

Cosmological constraints on the mass of Higgs boson can be obtained in other approaches like a consideration of the scalar in companion with another field as was done in [58], wherein the constraint in the form of $m_H < 134 \text{ GeV}$ has been derived.

In [59] a model of inflation was constructed in the framework of supergravity by means of setting an appropriate kind of Kahler potential with an additional symmetry keeping the Kahler potential to be independent of an imaginary part of scalar field. However, on a more deep level of superstring theory one cannot construct a way leading to models analogous to that of [59]. In [60] one has offered a realistic model of inflation in the framework of superstrings, so that problems of instability caused by the compactification of extra dimensions can be removed, and a vacuum is shifted from an initial anti-de Sitter state to a minimum of inflaton potential with a positive energy, i.e. to the de Sitter vacuum. Nevertheless, to our opinion, the development of inflation can occur at such densities of energy, whereat the supergravity is certainly broken, and a consistent field theory at the sub-Planckian energies should be based on the principle of renormalizability, while potentials derived in the framework of complete exact supergravity include terms of higher dimensions in the Newtonian constant G , which are not renormalizable. Therefore, such the terms presumably should be canceled after the breaking the supergravity, so that in the effective potential one has to hold the terms consistent with the requirement of renormalizability only, i.e. the terms being under control with respect to quantum loop-corrections. Otherwise, the field theory at energies below the Planck scale loses any predictable power. In this respect, one should to restrict himself by considering the supergravity corrections to the potential within limits of leading terms linear in the Newtonian constant G as was done by S. Weinberg in [61], combining this limit with the constraint on the degree of field self-action, that should be not greater than 4.

One can consider the problems of cosmological constant and inflationary dynamics beyond the general relativity, for instance, in the framework of conformal general relativity [62].

In the present paper we suggest to look at the problem of relaxing the cosmological constant and the naturalness of parameters in the potential of scalar field generating the inflation, from the unified point of view based on a phenomenological introduction of dynamical field with the flat potential $V = \text{const.}$ (up to contributions restricted by a power of inverse Planck mass), so that this potential value gives the primary cosmological constant. Then, the primary effective values of both the mass and self-action constant for the dynamical field are equal to zero. Such the fields can naturally appear in superstring theory and they are called modules: such the fields determine “flat directions” of effective potential. Then, we study whether the flat potential is stable under perturbations caused by

¹ Realistic models of low-energy inflation taking into account for constraints coming from the primordial nucleosynthesis, the anisotropy of CMBR and the inhomogeneity of matter distribution in the LSS of Universe are presented in [42–44], wherein the supersymmetric version of Standard Model is explored with the usage of flat directions in the superpotential.

quantum loop-corrections given by subleading terms in the inverse Planck mass.

So, we introduce such the field as a scalar component of chiral superfield in the supersymmetric theory. Further, we investigate the problem to determine nontrivial parameters of superpotential for the field, so that after taking into account for leading supergravity corrections in weak gravitational fields, i.e. *corrections linear in the gravitational constant*, the resulting potential would be flat, hence, the field would actually be the module. Such the procedure of determining the parameters of primary superpotential we call the cosmological bootstrap, because it fixes the initial mass and self-action constant of field in accordance with the requirement of canceling those terms by leading contributions due to the supergravity.

In this way, we assume that the scale of potential plateau is essentially less than the Planck mass, but it is large enough to agree with the concept of particle interactions: we put the scale to be close to an energy of grand unification of gauge interactions. Then, we find that the bootstrap is possible, i.e. the fine tuning can be specified, and the initial parameters of field are close to the mass and self-action constant for the inflaton, generating the observed large scale structure of Universe.

Next step of study is a small perturbations of fine tuning that leads to breaking of bootstrap for the mass and self-action of field as caused by higher corrections in the inverse Planck mass appearing due to loops taking into account for propagation of heavy particles with masses about the Planck mass. These quantum loop corrections result in the dynamical instability of primary cosmological constant, i.e. in the instability of initial flat potential, with the further relaxation of cosmological constant during the inflation generated by the field primary marked as the module. Then, the inflation parameters are naturally given by the breaking of bootstrap, and they agree with the observed values by the order of magnitude. It is important to emphasize that the mass, vacuum expectation value of the field as well as its self-action constant are actually determined by the introduction of single dimensional parameter being the scale of primary cosmological constant, which is natural for the particle physics.

II. BOOTSTRAP

In the theory of gravity with the dimensional coupling constant G it would be natural to expect that the vacuum energy ρ_G giving the cosmological constant, is determined by the Planck scale² $\tilde{m}_{\text{Pl}} = 1/\sqrt{8\pi G} \approx 2.4 \times 10^{18}$ GeV, so that

$$\rho_G = M^4, \quad (1)$$

where $M \sim \tilde{m}_{\text{Pl}}$. However, the supersymmetry can be dynamically related to another scale of energy Λ , which will be set much less than the Planck mass,

$$\Lambda \ll M. \quad (2)$$

In a phenomenological point of view, i.e. without a presentation of any mechanism for the introduction of scale Λ in addition to \tilde{m}_{Pl} , we will assume that in the model there are two dimensional quantities with the definite hierarchy of (2). In this way we will naturally suggest that the local supersymmetry, i.e. the supergravity leads to the vacuum energy equal to

$$\rho_S = \Lambda^4. \quad (3)$$

Such the finite renormalization³ from (1) to (3) is really possible in the supergravity, since the introduction of superpotential in the form of constant

$$W_0 = i\omega_0^3, \quad (4)$$

leads to the additional term⁴ in the density of vacuum energy, so that (see [61])

$$V_0 = \rho_G - 24\pi G |W_0|^2 = M^4 - 24\pi G \omega_0^6, \quad (5)$$

while the condition of $V_0 = \rho_S$ is the definition for the scale ω_0 . The above procedure presents the “nil” step of bootstrap being the consequent determination of theory parameters in terms of primary quantities, particularly, the

² We use the reduced Planck mass.

³ In this paper we do not consider questions on both a regularization of infinities and a renormalization group.

⁴ Following S. Weinberg [61], we restrict the consideration by corrections linear in the gravitational constant G .

determination of constant term in the superpotential in terms of two primary scales⁵ M and Λ . In this way, our goal is the description of primary cosmological constant as the flat potential.

Further, in the framework of supersymmetry the vacuum energy is determined by the potential

$$V_S = \left| \frac{\partial W}{\partial \Phi} \right|^2, \quad (6)$$

where, for the case of cosmological constant, i.e. for the potential independent of the field Φ , the superpotential should be written in the form

$$W \mapsto W_1 = i\omega_0^3 + \Lambda^2 \Phi, \quad (7)$$

so that

$$V_S = \Lambda^4. \quad (8)$$

The account for the supergravity with the linear terms in the Newtonian constant G leads to the potential of general form (see [61])

$$V = \rho_G + \left| \frac{\partial W}{\partial \Phi} \right|^2 - 24\pi G \left| W - \frac{1}{3} \Phi \frac{\partial W}{\partial \Phi} \right|^2 + \frac{16\pi G}{3} |\Phi|^2 \left| \frac{\partial W}{\partial \Phi} \right|^2. \quad (9)$$

Then the substitution of superpotential in the form of (7) into eq. (9) gives

$$V \mapsto V_1 = (M^4 - 24\pi G \omega_0^6) + \Lambda^4 - \frac{1}{2} \phi^2 \frac{16\pi G}{3} \Lambda^4, \quad (10)$$

if one puts the field $\phi = \Phi\sqrt{2}$ to be real (a consideration of general case is presented in Appendix I). Moreover, for the sake of simplicity in what follows, we suggest that the potential accounting for the supergravity corrections of (9), possesses the symmetry with respect to discrete operation of reflection $\phi \leftrightarrow -\phi$, i.e. it does not include odd degrees of field ϕ . At this step of bootstrap, the cosmological constant is given by the energy density V_S in (8), if we slightly adjust the parameter ω_0 , so that

$$M^4 - 24\pi G \omega_0^6 = 0. \quad (11)$$

However, in addition to the constant density of energy, the potential of (10) includes the term depending on the field. Moreover, the supergravity corrections leads to the instability of cosmological term. The instability actually signalizes that the supergravity renormalizes the term quadratic in the field, hence, the interaction quadratic in the field should be introduced at the stage of primary superpotential W . In addition, we see that in the theory with the global supersymmetry the module loses its main property in the theory with the local supersymmetry, i.e. in the supergravity. Therefore, we challenge the problem to determine a nontrivial superpotential, which describes the module after the account for the supergravity corrections in the form of (9).

Following such the bootstrap approach, we write down the superpotential with real parameters in the form

$$W = i\omega_0^3 + \Lambda^2 \Phi + \frac{i}{2} \mu_0 \Phi^2 + \frac{g_0}{3} \Phi^3. \quad (12)$$

Then, *with the accuracy up to terms quartic in the field*, i.e. accounting for the renormalizable terms, we get the potential $V = \hat{V} \Lambda^4$ with

$$\hat{V} = 1 - \frac{1}{2} \hat{\mu}^2 \hat{\phi}^2 + \frac{1}{4} \hat{\lambda} \hat{\phi}^4, \quad (13)$$

wherein we take into account the cancelation due to (11) at the “nil” step of bootstrap, and the following notations for the dimensionless quantities with hats are adapted:

$$\begin{aligned} \hat{\phi}^2 &= \frac{32\pi G}{3} |\Phi|^2, & \hat{g}_0 &= -\frac{3}{16\pi G \Lambda^2} g_0, \\ \hat{\omega}_0^6 &= \frac{16\pi G}{3\Lambda^4} \omega_0^6, & \hat{\mu}_0^2 &= \frac{3}{16\pi G \Lambda^4} \mu_0^2, \end{aligned} \quad (14)$$

⁵ Here we certainly suggest that all of quantities with the same order of magnitude are equivalent, of course.

so that

$$\begin{aligned}\hat{\mu}^2 &= 1 - \hat{\mu}_0^2 + 2\hat{g}_0 + \frac{3}{2}\hat{\omega}_0^3\hat{\mu}_0, \\ \hat{\lambda} &= \hat{g}_0^2 - 2\hat{g}_0 + \frac{7}{8}\hat{\mu}_0^2.\end{aligned}\tag{15}$$

Potential (13) represents the constant density of energy, if the bootstrap relations are in action,

$$\hat{\mu}^2 = \hat{\lambda} = 0.\tag{16}$$

Accounting for $\hat{\omega}_0^6 \gg 1$, we can easily find solutions of equations (16) for parameters $\hat{\mu}_0$ and \hat{g}_0 : there are two sets corresponding to solutions of quadratic equation for \hat{g}_0 ,

$$\hat{g}_0 = 1 \pm \sqrt{1 - \frac{7}{8}\hat{\mu}_0^2}.\tag{17}$$

Namely,

$$\begin{aligned}\hat{\mu}_0^I &\approx -\frac{2}{3}\frac{1}{\hat{\omega}_0^3}, & \hat{\mu}_0^{II} &\approx -\frac{10}{3}\frac{1}{\hat{\omega}_0^3}, \\ \hat{g}_0^I &\approx \frac{7}{16}\{\hat{\mu}_0^I\}^2, & \hat{g}_0^{II} &\approx 2 - \frac{7}{16}\{\hat{\mu}_0^{II}\}^2.\end{aligned}\tag{18}$$

Sub-leading corrections in (18) can be presented in the form of expansion in *even* degrees of ratio $\Lambda/\tilde{m}_{\text{Pl}}$. Since the coefficients of such the expansion are strictly definite, we can see that, first, the “fine tuning” of bootstrap parameters is required and, second, the mistuning leads to breaking of bootstrap. The mechanism of breaking we study in the present paper, is the following: the introduction of corrections in powers of $\Lambda/\tilde{m}_{\text{Pl}}$ to the “bare” superpotential causes the dynamical instability of primary cosmological constant, that leads to the inflationary expansion of Universe.

Sets (18) correspond to different hierarchies for the values of initial constant in the field self-action, \hat{g}_0 , since $\hat{g}_0^{II} \gg \hat{g}_0^I$, but they also lead to essentially different initial potentials of field in the framework of supersymmetry according to (6). Indeed, the primary potential $V_S = \hat{V}_S \Lambda^4$ can be written in the form

$$\hat{V}_S = 1 + \frac{\hat{\mu}_S^2}{2}\hat{\phi}^2 + \frac{\hat{\lambda}_S}{4}\hat{\phi}^4,\tag{19}$$

where

$$\begin{aligned}\hat{\mu}_S^2 &= \hat{\mu}_0^2 - 2\hat{g}_0, \\ \hat{\lambda}_S &= \hat{g}_0^2.\end{aligned}\tag{20}$$

For (18), the initial potential is stable: $\hat{g}_0^2 > 0$. Moreover, taking into account for the approximation $\hat{\omega}_0^6 \gg 1$, we can write down the explicit form

$$\begin{aligned}\hat{V}_S^I &\approx 1 + \left\{\frac{1}{4}\hat{\mu}_0^I\right\}^2\hat{\phi}^2 + \left\{\frac{7}{32}\hat{\mu}_0^I\right\}^2\hat{\phi}^4, \\ \hat{V}_S^{II} &\approx (1 - \hat{\phi}^2)^2 + \frac{1}{2}\{\hat{\mu}_0^{II}\}^2.\end{aligned}\tag{21}$$

The character of dependencies of initial potentials for the bootstrap sets I and II is shown in fig. 1, wherefrom we see that the set I corresponds to the situation, when the breaking of supersymmetry corresponds to the vacuum energy density Λ^4 , and the vacuum expectation value of field is equal to zero, while the set II involves the reduced value of field contribution to the supersymmetry breaking, that is accompanied by the spontaneous breaking of symmetry with respect to the discrete inversion $\phi \leftrightarrow -\phi$ suggested above. In addition, the set II is characterized by “natural” scaling for the values of both the mass and the constant of self-action, while for the set I these parameters take reduced values.

The breaking of bootstrap relations in (16) due to the quantum loop-corrections leads to instability of primary cosmological constant $\hat{V} = 1$ as well as to the inflation of Universe, if the minimum of potential corresponds to the scaled density of energy negligibly less than the unit. In this case, the primary cosmological constant relaxes from the huge value to that of we will put equal to zero⁶.

⁶ Otherwise, one should introduce the new dynamical field, which again causes the instability of remnant nonzero density of energy, if it is

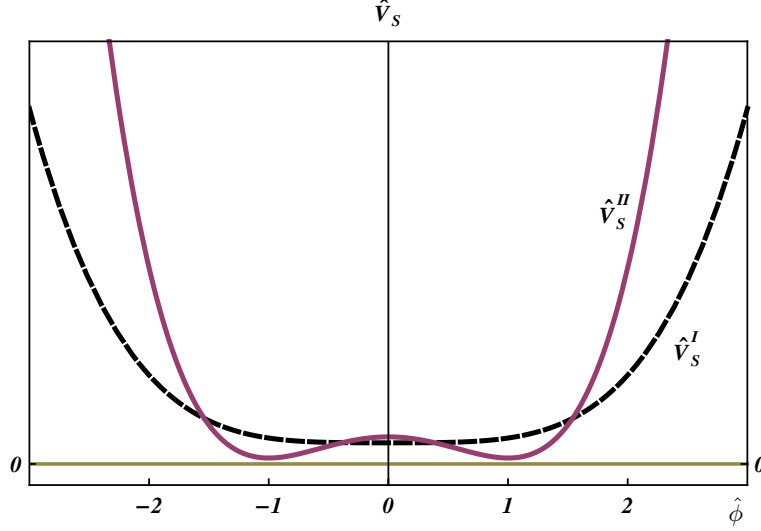


FIG. 1: The primary potential of scalar field for the sets I and II (in arbitrary units).

III. PHENOMENOLOGICAL ANALYSIS

The breaking of bootstrap relations is caused by the variation of parameters ω_0 , μ_0 and g_0 . In this section we do not specify any mechanism of breaking, but we will generally accept that the quantum corrections are expandable in the small ratio of $\Lambda/\tilde{m}_{\text{Pl}}$. In this way, we find some natural constraints.

First, corrections to ω_0 contribute to the vacuum energy, so that to the leading approximation it gets the contribution of the order of $\delta V_0 \sim \Lambda^4$, hence, $\delta\hat{\omega}_0^6 \sim \mathcal{O}(1)$, in accordance to the definition of $\delta V_0 \sim \Lambda^4 \delta\hat{\omega}_0^6$. By construction,

$$\hat{\omega}_0^3 \sim \left(\frac{\tilde{m}_{\text{Pl}}}{\Lambda}\right)^2 \Rightarrow \frac{\delta\hat{\omega}_0^3}{\hat{\omega}_0^3} \sim \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}}\right)^4. \quad (22)$$

The same correction can lead to the variation of parameter $\hat{\mu}_0 \sim \hat{\omega}_0^{-3}$, so that

$$\frac{\delta'\hat{\mu}_0}{\hat{\mu}_0} \sim \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}}\right)^4, \quad (23)$$

and the consequent variation of self-action constant $\delta'\hat{g}_0$ will appear, too, but such the transition of variations with the valid relations of bootstrap will not generally lead to the breaking of bootstrap itself, and we suggest that the parameters have got independent sources of corrections.

Second, the parameters of bootstrap have got the following orders of magnitude:

$$\hat{\mu}_0 \sim \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}}\right)^2, \quad \hat{g}_0 \sim \mathcal{O}(1) + \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}}\right)^4, \quad (24)$$

hence, the leading corrections can be written in the form

$$\delta\hat{\mu}_0 \sim \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}}\right)^{2+q}, \quad \delta\hat{g}_0 \sim \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}}\right)^{4+\tilde{q}}, \quad (25)$$

where the integer degrees are $q, \tilde{q} \geq 0$. The bootstrap breaking according to (15) is caused by nonzero values of quantities

$$\begin{aligned} \hat{\mu}^2 &= -2\hat{\mu}_0\delta\hat{\mu}_0 + 2\delta\hat{g}_0 + \frac{3}{2}\hat{\omega}_0^3\delta\hat{\mu}_0, \\ \hat{\lambda} &= 2(\hat{g}_0 - 1)\delta\hat{g}_0 + \frac{7}{4}\hat{\mu}_0\delta\hat{\mu}_0, \end{aligned} \quad (26)$$

positive. A negative value of remnant density of energy would lead to the collapse of Universe, that is not observed. See the discussion of this issue in the next section.

so that

$$\begin{aligned}\hat{\mu}^2 &\sim \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}}\right)^q + \mathcal{O}(1) \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}}\right)^{4+\tilde{q}}, \\ \hat{\lambda} &\sim \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}}\right)^{4+\tilde{q}} + \mathcal{O}(1) \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}}\right)^{4+q}.\end{aligned}\tag{27}$$

Then, the minimum of potential \hat{V} at $\hat{\phi}^2 = \hat{\mu}^2/\hat{\lambda}$ gets the value

$$\hat{V}_{\min} = 1 - \frac{\hat{\mu}^4}{4\hat{\lambda}},\tag{28}$$

and the relaxation of primary cosmological constant meaning the cancelation of contributions in the density of vacuum energy of the order of Λ^4 , is possible, if only

$$\frac{\hat{\mu}^4}{4\hat{\lambda}} \sim \mathcal{O}(1).\tag{29}$$

The condition of (29) implies that, in the case of $q \geq \tilde{q}$,

$$2q = 4 + \tilde{q},\tag{30}$$

so that there is the finite set of values for the correction degrees

$$\{q, \tilde{q}\} \mapsto \{2, 0\}, \{3, 2\}, \{4, 4\}.\tag{31}$$

If $q < \tilde{q}$, then there is the solution $q = 4$ with an arbitrary value of $\tilde{q} \geq 5$. In all of those cases, the scaling behavior of potential parameters is reduced to

$$\hat{\mu}^2 \sim \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}}\right)^q, \quad \hat{\lambda} \sim \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}}\right)^{2q},\tag{32}$$

where $q = \{2, 3, 4\}$.

Thus, we get the general phenomenological description of corrections responsible for the breaking of cosmological bootstrap. Remember, the corrections to the parameter $\hat{\omega}_0^6$ are reduced to the variation of $\delta\hat{V}_{\min} \sim \mathcal{O}(1)$, so that one gets the complete cancelation of terms in the vacuum density of energy of the order of Λ^4 , i.e. up to the required accuracy the condition $\hat{V}_{\min} = 0$ would be valid. This constraint is natural for the bootstrap construction, because the “survive” of terms like Λ^4 in the vacuum density of energy would lead to the necessity of additional introduction of secondary field of module, hence, if we consider the ultimate physical field of module, then the cancelation of cosmological term in the bootstrap breaking is simply the definition of such the field.

Finally, we adapt that the breaking of cosmological bootstrap for the field of module results in the scaling potential

$$\hat{V} = \left(1 - \frac{\hat{\mu}^2}{4} \hat{\phi}^2\right)^2.\tag{33}$$

Therefore, for the physical real field $\phi = \sqrt{2}|\Phi|$ the mass and constant of self-action have the forms

$$\begin{aligned}m^2 &= \frac{32\pi G}{3} \Lambda^4 \hat{\mu}^2, \\ \lambda &= \left(\frac{8\pi G}{3} \Lambda^2\right)^2 \hat{\mu}^4,\end{aligned}\tag{34}$$

while, by the order of magnitude,

$$\begin{aligned}m &\sim \tilde{m}_{\text{Pl}} \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}}\right)^{2+q/2}, \\ \lambda &\sim \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}}\right)^{4+2q},\end{aligned}\tag{35}$$

and the vacuum expectation value is equal to

$$v = \frac{m}{\sqrt{2\lambda}} \sim \tilde{m}_{\text{Pl}} \left(\frac{\tilde{m}_{\text{Pl}}}{\Lambda} \right)^{q/2}. \quad (36)$$

The inflaton mass is strictly constrained by the observational data⁷, so that, putting $m/\tilde{m}_{\text{Pl}} \sim 10^{-5}$, we get the following characteristic values of model parameters by the order of magnitude:

q	$\frac{\Lambda}{\tilde{m}_{\text{Pl}}}$	λ	$\frac{v}{\tilde{m}_{\text{Pl}}}$
2	2×10^{-2}	4×10^{-14}	50
3	4×10^{-2}	5×10^{-14}	150
4	6×10^{-2}	10^{-15}	300

The analysis of WMAP data on the anisotropy of cosmological microwave background radiation by 5 years of operation [7] in the framework of inflaton with the Higgsian kind of potential [63] leads to the following constraints on the vacuum expectation value of the field:

- in the new scenario of inflation (the field slowly rolling down to the minimum from the state close to the instable local maximum at $\phi = 0$, the “hilltop” inflation)

$$\frac{v}{\tilde{m}_{\text{Pl}}} \geq 10, \quad (37)$$

- in the scenario of chaotic inflation (the evolution of field to the minimum from $|\phi| > v$)

$$\frac{v}{\tilde{m}_{\text{Pl}}} \geq 100, \quad (38)$$

so that among the admissible values of parameter characterizing the contribution of corrections we have to prefer for $q = 2$, since the large vacuum expectation values correspond to the potential, which degenerates to the limit of $V \sim \phi^2$, that actually lies at the edge of region with the 1σ -confidence level. At $q = 2$, the scale of primary cosmological constant is $\Lambda \sim 5 \times 10^{16}$ GeV, that, in fact, is consistent with the suggestion on the scale, corresponding to the grand unification of gauge interactions.

This point gets a more significant confirmation after the account of WMAP data during the 7-years operation [9], which exclude the chaotic scenario of inflation (with $|\phi| > v$) at the 1σ -confidence level and give the followings:

- the mass of inflaton equals

$$m_{\text{inf}} \approx (1.30 - 1.74) \times 10^{13} \text{ GeV}, \quad (39)$$

- the vacuum expectation value $\langle \phi \rangle = v$ is constrained by

$$2.5 m_{\text{Pl}} < v < 54 m_{\text{Pl}}, \quad (40)$$

where $m_{\text{Pl}} = \sqrt{8\pi} \tilde{m}_{\text{Pl}}$ is the Planck mass,

- the “hilltop” inflation is actual, only, the fields rolls from the plateau near the “nil” value of field to the potential minimum, while the plateau magnitude is given by the parameter $V(0) = V_{\text{hill}} = \Lambda_{\text{hill}}^4$,

$$\Lambda_{\text{hill}} = (1.2 - 6.0) \times 10^{16} \text{ GeV}. \quad (41)$$

The analysis is shown in fig. 2, wherefrom we see that the preferable value is $q = 2$.

Thus, we expect that in the viable model of corrections, the parameters of primary superpotential are shifted in accordance with the rules

$$\delta\mu_0^2 \sim -\mu_0^2 \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}} \right)^2, \quad \delta g_0 \sim \pm \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}} \right)^6, \quad (42)$$

where the sign of δg_0 corresponds to the stability of potential ($\tilde{\lambda} > 0$) for the sets I and II, respectively.

⁷ We follow the analysis performed by us in the method of driftage of attractor in the phase plain, that describes the inflationary dynamics [63–65].

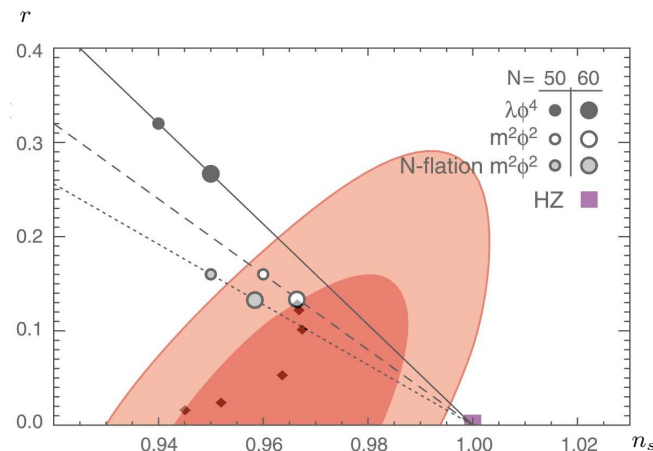


FIG. 2: The comparison of data observed with the 1σ and 2σ accuracy for the spectral index of scalar fluctuations of energy density n_s and relative contribution of tensor fluctuations r from [9] with predictions of inflation models for the quadratic and quartic self-action, and the model with N scalar fields (N-flation), as well as with the limit of scale invariant fluctuations by Harrison–Zeldovich (HZ) at the horizon, shifted from the end of inflation by e-foldings of scale factor, $N = 50$ and $N = 60$. The predictions of new scenario of inflation are shown by rhombuses at $N = 60$ and vacuum expectation values of inflaton $v/m_{\text{Pl}} = 2.5, 2.8, 4, 7.4, 24.6, 54$ (in course of growing r).

IV. 1-LOOP STRUCTURE

In the simplest case, after the supersymmetry breaking the model includes the single light scalar real field, whereas the notion “light” implies that the field mass is essentially less than the scale of primary cosmological constant:

$$m \ll \Lambda.$$

Moreover, the light fields are the massless graviton and the gravitino, whose mass is given by the following formula derived in the leading approximation [61]:

$$m_g^2 = \frac{8\pi G}{3} \Lambda^4 \ll \Lambda^2. \quad (43)$$

To the low-energy approximation the heavy fields of inflatino and imaginary part of scalar field do not propagate. The propagators and vertices of interactions are presented in Appendix III. They are consistent with the superpotential and supercurrent of chiral superfield.

Then, the 1-loop terms in the effective potential of inflaton occur due to the loops of

1. the inflaton field itself,
2. the gravitino from the supercurrent with the inflatino, which propagator is reduced to a constant at low energies,
3. the graviton.

In this section we analyze the model with the chiral superfield under the regularization in the Euclidean space by means of momentum cut-off in the loop.

A. Generation of μ_0

It is interesting to notice that the contraction of inflatino propagator into the point at low energies due to Planckian scale of inflatino mass m' leads to a natural introduction of mass parameter in the superpotential, μ_0 initially equal to zero. Indeed, the vertexes for the coupling of real scalar field to the inflatino due to the self-action constant g_0 and for the interaction of inflatino with the gravitino due to the vacuum energy at the scale Λ are effectively reduced to the vertex for the coupling of scalar field to the inflatino and gravitino (see fig. 3)⁸, hence, one generates μ_0 in the

⁸ We take into account for both the chiral rotation for the inflatino (see Appendix II) and the Feynman rules (see Appendix III).

form

$$\mu_0 = 2g_0 \frac{\Lambda^2}{m'}. \quad (44)$$

Accounting for the scale of inflatino mass $m' \sim \tilde{m}_{\text{Pl}}$, we straightforwardly see that the cosmological bootstrap corresponds to the situation, when $g_0 \sim (\Lambda/\tilde{m}_{\text{Pl}})^2$ and, hence, $\mu_0 \sim \tilde{m}_{\text{Pl}}(\Lambda/\tilde{m}_{\text{Pl}})^4$, as it would be for the solution with the set II.

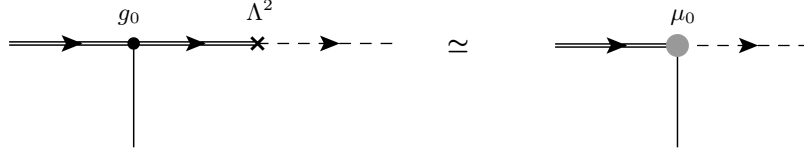


FIG. 3: The effective vertex for the coupling of inflaton to the inflatino and gravitino after the contraction of inflatino propagator into the point; the solid line denotes the propagator for the inflaton, the double line does for the inflatino, the dashed line gives the gravitino; the vertex for the coupling of inflatino to the inflaton is marked by the bold dot, while the vertex for the transition of inflatino to the gravitino is denoted by the cross.

Then, the initial bare superpotential with zero value of μ_0 leads to the potential $V_S^0 = \Lambda^4(\hat{\phi}^2 - 1)^2$ [see (21)], i.e. it corresponds to *zero energy of vacuum*. Evidently, the generation of effective mass parameter μ_0 at low energy as described above occurs at the tree level, hence it does not change the *vacuum energy*, which remains *equal to zero*.

The variation of bare value (44) under the introduction of corrections has got two following sources: δg_0 and $\delta \Lambda^2$, so that

$$\delta \mu_0 = \mu_0 \left(\frac{\delta g_0}{g_0} + \frac{\delta \Lambda^2}{\Lambda^2} \right) \approx \mu_0 \frac{\delta \Lambda^2}{\Lambda^2}, \quad (45)$$

where the approximation has been accepted in accordance with the set II, when the relative contribution by the variation of self-coupling constant g_0 is suppressed. Therefore, following (42) and (45), we expect the correction of the form

$$\frac{\delta \Lambda^2}{\Lambda^2} \sim - \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}} \right)^2. \quad (46)$$

Note, in the case of cosmological bootstrap, relation (44) means that there is the connection between the mass of inflatino with the primary density of vacuum energy of Planckian scale, since the parameter μ_0 in set II is determined by the quantity ω_0 [see (18)] and its value at the first step of bootstrap [see (11)], i.e. by the cancelation of Planckian contributions to the vacuum energy, so that up to small corrections we find

$$m' = \frac{16}{5\sqrt{6}} M^2 \sqrt{\pi G} \sim \tilde{m}_{\text{Pl}}. \quad (47)$$

This relation corresponds to the requirement of existing the flat potential of module, and we do not consider this connection as the condition of any “fine tuning” for the model parameters, but, presumably, we put it as the definition of the module itself, i.e. as the prerequisite of our model.

B. Zero-point modes of field and regularization

A free real scalar-field has got the canonical tensor of energy-momentum

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} (\partial_\alpha \phi \partial^\alpha \phi - m^2 \phi^2), \quad (48)$$

and zero-point modes of field contribute due to the imaginary part of propagator in the loop [see fig. 4]:

$$\langle T_{\mu\nu} \rangle_0 = \int \frac{d^4 p}{(2\pi)^4} \left\{ p_\mu p_\nu - \frac{1}{2} g_{\mu\nu} (p^2 - m^2) \right\} (i)^2 \Im \frac{1}{p^2 - m^2 + i0}. \quad (49)$$

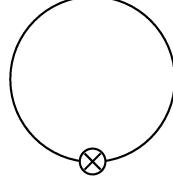


FIG. 4: The vacuum loop of real scalar field with the insertion of operator $p_\mu p_\nu - \frac{1}{2} g_{\mu\nu}(p^2 - m^2)$.

Taking into account for

$$\Im \frac{1}{p^2 - m^2 + i0} = -\pi \delta(p^2 - m^2), \quad (p^2 - m^2) \delta(p^2 - m^2) = 0,$$

we find that the contribution to the vacuum expectation of energy-momentum tensor due to the field lagrangian with the factor of metric nullifies⁹, while in the Minkowskian space the formula

$$\langle T_{\mu\nu} \rangle_0 = \int \frac{d^4 p}{(2\pi)^4} p_\mu p_\nu \pi \delta(p^2 - m^2) \quad (50)$$

is reduced to the standard form for the zero-point modes after the integration out of the delta-function over the temporal component of momentum

$$\langle T_{\mu\nu} \rangle_0 = \pi \int \frac{d^3 p}{(2\pi)^4 2|p_0|} p_\mu p_\nu \Big|_{p_0 = \pm |p_0|}. \quad (51)$$

This formula takes the sense under a regularization, for instance, due to the cut off the spatial momentum by the scale Λ_M . Then, because of the spherical symmetry the resulting averaged energy-momentum tensor corresponds to an ultra-relativistic matter in the case, when the cut-off scale is essentially greater than the field mass, $\Lambda_M \gg m$, or to a non-relativistic matter, “dust”, if the mass is essentially greater than the cut-off $m \gg \Lambda_M$. Really,

$$\langle T_{00} \rangle_0 = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{2} |p_0|, \quad \langle T_{\alpha 0} \rangle_0 = \langle T_{0\alpha} \rangle_0 = 0,$$

$$\langle T_{\alpha\beta} \rangle_0 = \int \frac{d^3 p}{(2\pi)^3 2|p_0|} p_\alpha p_\beta = \frac{1}{3} \delta_{\alpha\beta} \int \frac{d^3 p}{(2\pi)^3 2|p_0|} p^2.$$

However, such the result seems to be not physical, because such the zero-point modes possess the energy-momentum tensor different from the vacuum one, that should be proportional to the metric. This fact points to that a regularization should be consistent with the physical requirements (see [66]). The trivial condition would be the introduction of normal order for operators in the definition of energy-momentum tensor, that simply puts the contribution of zero-point modes to be equal to zero. We go in another way, introducing the regularization in the Euclidean space after the Wick rotation: $p_0 = ip_4$, so that the expression for the contribution by the zero-point modes into the averaged tensor of energy-momentum becomes equal to

$$\langle T_{\mu\nu} \rangle_0^E = \int \frac{i d^4 p_E}{(2\pi)^4} \frac{-i}{p_E^2 + m^2} p_\mu^E p_\nu^E = \frac{1}{4} g_{\mu\nu}^E \int \frac{d^4 p_E}{(2\pi)^4} \frac{p_E^2}{p_E^2 + m^2}. \quad (52)$$

where we have taken into account for the spherical symmetry of Euclidean space. Thus, the contribution of single zero-point mode of real scalar field gives

$$\langle T_{\mu\nu} \rangle_0^E = -g_{\mu\nu} \frac{1}{(16\pi)^2} \left\{ \frac{1}{2} p_E^4 - m^2 p_E^2 + m^4 \ln \frac{p_E^2 + m^2}{m^2} \right\} \Big|_{\Lambda_d^2}^{\Lambda_u^2}, \quad (53)$$

⁹ Generally, this term represents the classical expression giving the value of potential in its minimum, that is equal to zero for the free field, of course.

where $\Lambda_{u,d}$ denote the upper and lower boarders in the cut-off over the Euclidean momentum, respectively.

For the Majorana fermion with the energy-momentum tensor

$$T'_{\mu\nu} = \frac{1}{2} \bar{\psi} p_\mu \gamma_\nu \psi - \frac{1}{2} g_{\mu\nu} \bar{\psi} (\hat{p} - m') \psi, \quad (54)$$

the analogous procedure taking into account for the elementary calculation of trace for Dirac's gamma-matrices $\text{tr}[\gamma_\nu (\hat{p} + m')] = 4p_\nu$ and for the minus sign in the case of fermionic loop, leads to the following expression for the contribution of zero-point modes:

$$\langle T'_{\mu\nu} \rangle_0^E = g_{\mu\nu} \frac{2}{(16\pi)^2} \left\{ \frac{1}{2} p_E^4 - m'^2 p_E^2 + m'^4 \ln \frac{p_E^2 + m'^2}{m'^2} \right\} \Bigg|_{\Lambda_d^2}^{\Lambda_u^2}, \quad (55)$$

where factor 2 corresponds to two, left and right modes of Majorana particle in comparison with the case of scalar field.

Summing up the contributions by components of chiral superfield into the energy-momentum tensor of vacuum gives zero, if the exact supersymmetry takes place (all of masses in the supermultiplet are equal to each other), while after the breaking down the supersymmetry the sum rules

$$\sum (-1)^F = 0, \quad \sum (-1)^F m^2 = 0,$$

lead to the expression¹⁰

$$\langle T_{\mu\nu} \rangle^E = g_{\mu\nu} \frac{1}{(16\pi)^2} \left\{ 2m'^4 \ln \frac{p_E^2 + m'^2}{m'^2} - \tilde{m}^4 \ln \frac{p_E^2 + \tilde{m}^2}{\tilde{m}^2} \right\} \Bigg|_{\Lambda_d^2}^{\Lambda_u^2}, \quad (56)$$

where we have neglected the contribution by the real part of scalar field, because of approximating its mass equal to zero. Therefore, the sum rules give $\tilde{m}^2 = 2m'^2$, and finally, we get

$$\begin{aligned} \langle T_{\mu\nu} \rangle^E &= g_{\mu\nu} \frac{2}{(16\pi)^2} \left\{ m'^4 \ln \frac{p_E^2 + m'^2}{m'^2} - 2m'^4 \ln \frac{p_E^2 + 2m'^2}{2m'^2} \right\} \Bigg|_{\Lambda_d^2}^{\Lambda_u^2}, \\ &= g_{\mu\nu} \frac{2m'^4}{(16\pi)^2} \ln \frac{1 + \frac{p_E^2}{m'^2}}{\left(1 + \frac{p_E^2}{2m'^2}\right)^2} \Bigg|_{\Lambda_d^2}^{\Lambda_u^2}. \end{aligned} \quad (57)$$

Since the logarithm in formula (57) has got negative values, we can draw the following conclusions:

- if the upper limit of integration has got values of the order of Planckian scale, $\Lambda_u \sim \tilde{m}_{\text{Pl}}$, then it gives the negative contribution into the density of vacuum energy, that corresponds to the cancelation of initial value of vacuum energy M^4 due to the introduction of suitable value of parameter ω_0 in the superpotential;
- if the lower limit satisfies the condition $\Lambda_d \ll m'$, then we get

$$\langle T_{\mu\nu} \rangle_u^E = g_{\mu\nu} \frac{1}{(16\pi)^2} \frac{1}{2} \Lambda_d^4 \sim g_{\mu\nu} \Lambda^4, \quad (58)$$

that corresponds to the introduction of vacuum energy of the order of Λ^4 at $\Lambda_d \sim \Lambda$.

Thus, the integration over the Euclidean momentum squared in limits $[\Lambda_E^2, M_E^2]$ at $\Lambda_E \sim \Lambda$ and $M_E \sim \tilde{m}_{\text{Pl}}$ forms the initial conditions for the introduction of chiral superfield of module. The interval of integration $[0, \Lambda_E]$, i.e. the regularization in the Euclidean space with the cut-off Λ_E , corresponds to the contribution at low energies due to

¹⁰ Here \tilde{m} denotes the mass of imaginary part of scalar filed.

the light fields. In what follows, we will consider similar intervals of integrations in the calculation of loops for the effective potential.

Then, the correction to the energy-momentum tensor of vacuum due to the zero-point modes gets the form¹¹

$$\delta T_{\mu\nu}^E = -g_{\mu\nu} \frac{1}{(16\pi)^2} \frac{1}{2} \Lambda_E^4. \quad (59)$$

It is important to note that the separation of Planckian interval of integration with the introduction of intermediate cut-off Λ_E as given above, implies that we get the exact cancelation of vacuum energy of the order of Λ^4 ! Moreover, we clearly see that the switching on the field interactions with the coupling constants in the form of power corrections in $\Lambda/\tilde{m}_{\text{Pl}}$ does not change our statements.

C. Generation of g_0

While the origin of mass parameter μ_0 can be quite naturally explained by the introduction of effective constant due to the contraction of inflatino propagator in the case of non-zero self-action of the field, the appearance of self-action itself with the coupling constant g_0 , corresponding to the cosmological bootstrap, is not trivial, because it is related to the existence of primary module field. However, we can clarify this issue by the following note: the modification of canonical energy-momentum tensor of free real scalar field by the formula¹²

$$T_{\mu\nu}^{\text{mod}} = \phi p_\mu p_\nu \phi - \frac{1}{2} g_{\mu\nu} \phi(p^2 - m^2)\phi, \quad (60)$$

due to the account for the interaction with the graviton, leads to quadratic term of scalar field interaction in the effective potential (see the diagram in fig. 5).



FIG. 5: The gravitational correction to the quadratic term in the effective potential of inflaton (the wave line denotes the graviton).

Indeed, the calculation of loop in the Euclidean space up to the leading approximation¹³ gives

$$-i 2g_0 \Lambda^2 = \int \frac{i d^4 p_E}{(2\pi)^4} 4\pi G \Rightarrow g_0 = -\frac{G}{16\pi} \frac{c_g \Lambda_E^4}{\Lambda^2}, \quad (61)$$

where we have introduced the dimensionless constant c_g in order to parameterize the arbitrary choice of ultraviolet cut-off in agreement to the procedure of renormalizations. In this way, we suggest that c_g is of the order of unit. Therefore, first, the generated value of bare constant g_0 has got the desired order in the ratio $\Lambda/\tilde{m}_{\text{Pl}}$, and second, it can get the required value for the leading approximation in the bootstrap, if we put the cut-off equal to

$$c_g \Lambda_E^4 = \frac{2}{3} (16\pi \Lambda^2)^2 \Rightarrow g_0 = -\frac{32\pi G}{3} \Lambda^2. \quad (62)$$

In this case, the correction to the vacuum energy due to the loop of zero-point modes has got the form

$$\delta T_{\mu\nu}^E = -\frac{1}{3c_g} g_{\mu\nu} \Lambda^4, \quad (63)$$

¹¹ This correction certainly corresponds to the leading contribution of real scalar field being the only component of chiral superfield that possesses the mass essentially less than the cut-off; at such the cut-off the “heavy” field components give small corrections to $\delta T_{\mu\nu}^E$ of the order of $(\Lambda/\tilde{m}_{\text{Pl}})^2$.

¹² The momentum operator acts to one of scalar fields, for instance, to the right one. Of course, we can write down the expression in terms of partial derivatives, which symmetrically act twice to the field in the right and to the field in the left.

¹³ Here, we neglect the contribution by the mass term.

that show the necessity of contribution by other fields into the vacuum energy of the order of Λ^4 , if $c_g \neq \frac{1}{3}$. In this respect, we have to note that we have suggested that the loop under the consideration gives the contribution to the constant of quadratic self-action of field, while the field normalization remains unchanged, though generically one has to take into account for the opportunity of such the renormalization in powers of $\Lambda/\tilde{m}_{\text{Pl}}$.

The above study shows that primary parameters of model can be generated due to the introduction of loop corrections with virtual gravitons and, hence, gravitinos to the free scalar field, if the cut-off is set by the scale of primary cosmological constant Λ , by the order of magnitude, of course. However, it means that loops with gravitons and gravitinos should not be taken into account with the same cut-off, while we consider the breaking of cosmological bootstrap, or we have to put the corresponding cut-off in loops with gravitons and gravitinos to be suppressed by (even) degrees in $\Lambda/\tilde{m}_{\text{Pl}}$, otherwise we would not distinguish the leading order from the corrections. The same conclusion can be drawn, if we consider the contributions of zero-point modes of graviton and gravitinos into the vacuum energy: at the cut-off taken of the order of Λ such the modes occur inadmissibly large, i.e. they get the same order of Λ^4 as the matter fields, although we would expect that the gravitational field contribute as corrections to the vacuum energy suppressed by the small ratio $(\Lambda/\tilde{m}_{\text{Pl}})^2$, at least. Further we will convince that the model of cosmological bootstrap would be reasonable, if only one-loop contributions by gravitons and gravitinos have got the cut-offs with the mentioned suppression in $\Lambda/\tilde{m}_{\text{Pl}}$.

D. Inflaton loops

The one-loop corrections caused by the real scalar field only, corresponds to contributions into the effective potential as shown in diagrams represented in fig. 6, i.e. they give the corrections to the field self-action with degrees of 2 and 4. The amplitudes are given by expressions

$$-iL_1 = -3 \frac{g_0^2}{16\pi^2} i c_2 \Lambda_E^2, \quad -iL_2 = 54 \frac{g_0^4}{16\pi^2} i \ln \frac{c_4 \Lambda_E^2}{\Lambda_{\text{reg}}^2}, \quad (64)$$

corresponding to the corrections to the lagrangian

$$\delta_1 \mathcal{L} = \frac{1}{2} L_1 \phi^2 + \frac{1}{4!} L_2 \phi^4,$$

whereas in (64) we have introduced the normalization scale of logarithmic corrections Λ_{reg} as well as the dimensionless constants $c_{2,4}$, corresponding to variations of cut-off parameter for different physical quantities in the regularization and renormalization.

Then, L_2 provides us with the standard renormalization of constant g_0 :

$$\delta g_0 = -9 \frac{g_0^3}{32\pi^2} \ln \frac{c_4 \Lambda_E^2}{\Lambda_{\text{reg}}^2} \Rightarrow \frac{dg_0}{d \ln \Lambda_{\text{reg}}} = 9 \frac{g_0^3}{16\pi^2}, \quad (65)$$

so that at $\Lambda_{\text{reg}} < \Lambda_E$ and $g_0 < 0$ it leads to

$$\delta g_0 \sim \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}} \right)^6 > 0,$$

as we could expect in the bootstrap model, set II (see (42)).

Further, L_1 can be naturally treated as the correction to the primary value of $2g_0\Lambda^2$, determining the quadratic self-action of field, whereas, because the variation of g_0 has got the large degree in small $\Lambda/\tilde{m}_{\text{Pl}}$ as found, we have to put

$$L_1 = 2g_0\delta\Lambda^2,$$

resulting in

$$\delta\Lambda^2 = \frac{3}{2} \frac{g_0}{16\pi^2} c_2 \Lambda_E^2 < 0. \quad (66)$$

In accordance to (45) we get

$$\frac{\delta\mu_0}{\mu_0} = \frac{3}{2} \frac{g_0}{16\pi^2} \frac{c_2 \Lambda_E^2}{\Lambda^2} \sim - \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}} \right)^2 < 0. \quad (67)$$

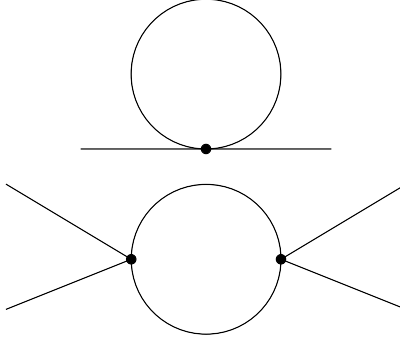


FIG. 6: The 1-loop corrections to the effective potential of inflaton: the self-action of second and fourth degrees in the field.

Let us redefine the dimensionless quantities with hats in (14) by the substitution of $\Lambda^2 \mapsto \Lambda^2 + \delta\Lambda^2$. Such the procedure gives the potential of scalar field in the form

$$V = (\Lambda^2 + \delta\Lambda^2)^2 \left(1 - \frac{\hat{\mu}^2}{2} \hat{\phi}^2 + \frac{\hat{\lambda}}{4} \hat{\phi}^4 \right), \quad (68)$$

where

$$\begin{aligned} \hat{\mu}^2 &= -5 \frac{\delta\mu_0}{\mu_0} = -\frac{15}{2} \frac{g_0}{16\pi^2} \frac{c_2 \Lambda_E^2}{\Lambda^2}, \\ \hat{\lambda} &= 4 \frac{\delta g_0}{g_0} = -9 \frac{g_0^2}{4\pi^2} \ln \frac{\Lambda_E \sqrt{c_4}}{\Lambda_{\text{reg}}}, \end{aligned} \quad (69)$$

and we evidently see that we can easily chose a reasonable value of renormalization point Λ_{reg} , which would give zero value of vacuum energy density: $\hat{\mu}^4 = 4\hat{\lambda}$. At this condition, the normalization point becomes close to the vacuum expectation value of field itself;

$$\langle \phi \rangle^2 = \frac{3}{16\pi G} \langle \hat{\phi} \rangle^2 = \frac{3}{4\pi G \hat{\mu}^2} \sim \Lambda_{\text{reg}}^2. \quad (70)$$

Thus, the considered contributions provide us with the natural realization of cosmological bootstrap.

However, loops of scalar field also have lead to corrections for vertexes of contact interaction between the inflaton, gravitino and inflatino due to the introduction of supercurrent, so that these loops are analogous to those of calculated above. In addition, we have to emphasize that at low energies inflatino with the mass of Planckian scale is “frozen”, hence, external inflatino fields are equal to zero, and therefore, loops of scalar fields in the mentioned vertexes of supercurrent have no relation to the considered contributions, because external inflatino-legs in the diagrams lead to zero values of those corrections.

E. Loops of gravitino and gravitons

To find the correction quadratic in the inflaton field let us consider the loop diagram¹⁴ with the gravitino and inflatino (fig. 7) in accordance with the Feynman rules (see Appendix III), wherein we put the incoming and outcoming momenta of inflaton to be equal to zero, while the loop momentum is denoted by k . Then

$$\begin{aligned} \delta\mu^2 &= -\frac{\mu_0^2(8\pi G)}{4} \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left\{ \frac{P(k)^{\mu\nu}}{k^2 + m_g^2} \gamma_\nu \gamma^5 \frac{-i\gamma k + m'}{k^2 + m'^2} \gamma_\mu \gamma^5 \right\} \\ &= -\frac{(8\pi G)\mu_0^2 c_m \Lambda_E^4}{24\pi^2 m_g} \left(\frac{1}{m'} + \mathcal{O}\left(\frac{1}{m'^2}\right) \right). \end{aligned} \quad (71)$$

¹⁴ In this section all of integrals are calculated in the Euclidean space.

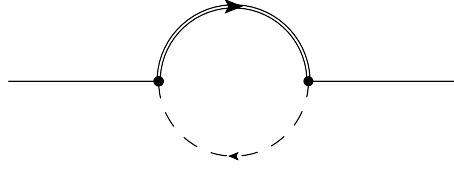


FIG. 7: The contribution of loop with the inflatino into the quadratic self-action of inflaton.

Here we introduce a constant c_m as the parameter of ultraviolet cut-off. Since the gravitino propagate in the loop, we have to put $c_m \ll 1$, so that taking into account for the expression giving the gravitino mass, we get that the contribution under study is suppressed as

$$\frac{\delta\mu^2}{\mu_0^2} \sim -c_m \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}} \right)^2,$$

and it is not essential in respect of consideration for the breaking down the cosmological bootstrap.

The correction to g_0 are given by two diagrams with zero outgoing momenta and the loop momentum k (fig. 8).

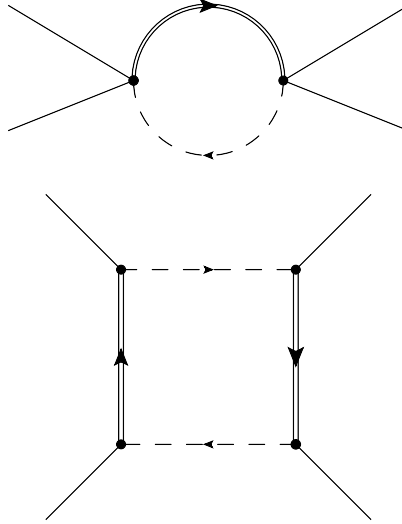


FIG. 8: The contribution of loops with the inflatino into the quartic self-action of inflaton.

In accordance with the first diagram in fig. 8 we get

$$\begin{aligned} g_0 \delta g_0 &= \frac{3}{12} \frac{g_0^2 (8\pi G)}{2} \int \frac{d^4 k}{(2\pi)^4} \text{Tr} \left\{ \frac{P(k)^{\mu\nu}}{k^2 + m_g^2} \gamma_\nu \frac{-i\gamma k + m'}{k^2 + m'^2} \gamma_\mu \right\} \\ &= -\frac{(8\pi G) g_0^2 c'_g \Lambda_E^4}{48\pi^2 m_g} \left(\frac{1}{m'} + \mathcal{O}\left(\frac{1}{m'^2}\right) \right) \sim -g_0^2 c'_g \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}} \right)^2. \end{aligned} \quad (72)$$

The second diagram in fig. 8 results in the following analytic expression:

$$\begin{aligned} g_0 \delta g_0 &= \frac{6}{12} \frac{\mu_0^4 (8\pi G)^2}{16} \int \frac{d^4 k}{(2\pi)^4} \times \\ &\quad \text{Tr} \left\{ \frac{P(k)^{\mu\nu}}{k^2 + m_g^2} \gamma_\nu \gamma^5 \frac{-i\gamma k + m'}{k^2 + m'^2} \gamma_\sigma \gamma^5 \frac{P(k)^{\sigma\rho}}{k^2 + m_g^2} \gamma_\rho \gamma^5 \frac{-i\gamma k + m'}{k^2 + m'^2} \gamma_\mu \gamma^5 \right\} \\ &\approx \frac{\mu_0^4 (8\pi G)^2 c''_g \Lambda_E^2}{8m'^2} + \mathcal{O}\left(\frac{1}{m'^3}\right) \sim -g_0 c''_g \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}} \right)^{12}. \end{aligned} \quad (73)$$

From (72) we see that, in the presence of gravitino within the loop, the constant of cut-off should be suppressed as $c'_g \sim (\Lambda/\tilde{m}_{\text{Pl}})^2$. Then, the model of cosmological bootstrap remains consistent, though the inflatino gives an essential contribution into variation of field self-action as large as comparable to the logarithmic renormalization, and probably, it plays a dominant role. Nevertheless, this fact does not change estimates in the order of magnitude as was done in previous sections. From (73) we see that the role of constant c'_g is inessential.

Corrections caused by the interaction of canonical tensor of energy-momentum with the graviton are given by diagrams in fig. 9.

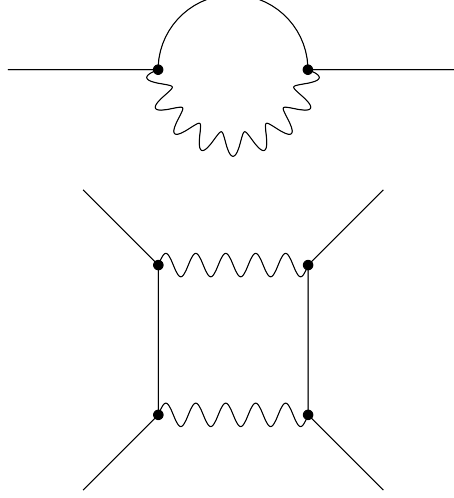


FIG. 9: The contribution of loops with gravitons (waved line) into the self-action of inflaton.

The first diagram in fig. 9 gives the logarithmic correction to the mass in the form of

$$\begin{aligned} \delta\mu^2 &= -(8\pi G)(\mu_0^2 + 2g_0\Lambda^2)^2 \eta_{\mu\nu}(\eta^{\mu\mu'}\eta^{\nu\nu'} + \eta^{\mu\nu'}\eta^{\nu\mu'} - \eta^{\mu\nu}\eta^{\mu'\nu'})\eta_{\mu'\nu'} \times \\ &\quad \int \frac{d^4k}{(2\pi)^4} \frac{1}{2k^2} \frac{1}{k^2} = \frac{(8\pi G)(\mu_0^2 + 2g_0\Lambda^2)^2}{4\pi^2} \ln \frac{c'_m \Lambda_E^2}{\Lambda_{\text{reg}}^2} \\ &\sim \Lambda^2 \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}} \right)^6 \ln \frac{c'_m \Lambda_E^2}{\Lambda_{\text{reg}}^2}, \end{aligned} \quad (74)$$

while the second diagram in fig. 9 does the correction to the constant of self-action

$$\begin{aligned} g_0 \delta g_0 &= -\frac{6}{12} (8\pi G)^2 (\mu_0^2 + 2g_0\Lambda^2)^4 \eta_{\mu\nu}(\eta^{\mu\mu'}\eta^{\nu\nu'} + \eta^{\mu\nu'}\eta^{\nu\mu'} - \eta^{\mu\nu}\eta^{\mu'\nu'})\eta_{\mu'\nu'} \times \\ &\quad \eta_{\mu_1\nu_1}(\eta^{\mu_1\mu'_1}\eta^{\nu_1\nu'_1} + \eta^{\mu_1\nu'_1}\eta^{\nu_1\mu'_1} - \eta^{\mu_1\nu_1}\eta^{\mu'_1\nu'_1})\eta_{\mu'_1\nu'_1} \int \frac{d^4k}{(2\pi)^4} \frac{1}{4k^4} \frac{1}{(k^2)^2} \\ &= \frac{(8\pi G)^2 (\mu_0^2 + 2g_0\Lambda^2)^4}{4\pi^2} \left\{ \frac{1}{\tilde{c}_g \Lambda_E^4} - \frac{1}{\Lambda_{\text{reg}}^4} \right\} \sim -g_0 \frac{1}{\tilde{c}_g} \left(\frac{\Lambda}{\tilde{m}_{\text{Pl}}} \right)^{10}. \end{aligned} \quad (75)$$

Contribution (74) can be treated as the suppressed logarithmic correction to the scale Λ^2 like $g_0\delta\Lambda^2$, that is inessential, or as the variation of δg_0 , which can be comparable with the logarithmic correction due to the loop with the inflaton calculated above. In the second case, in accordance with $c'_m \sim \Lambda/\tilde{m}_{\text{Pl}}$ we find that the logarithm argument has got an additional power in the small parameter, though this fact makes it value to be of the same order as the renormalization correction. In this way, the correction in (74) has got the opposite sign, so that one can get a compensation of contributions. Probably, this fact points that the main contribution is given by (72).

Following (75) we conclude that even at $\tilde{c}_g \sim (\Lambda/\tilde{m}_{\text{Pl}})^2$ this contribution is not essential for our consideration.

Thus, we have shown that the one-loop structure of theory is consistent with relations required for the reasonable breaking down the cosmological bootstrap. It means that the instability of primary cosmological constant is matched to the inflation of Universe.

V. CONCLUSION

We have confirmed the possibility to construct the realistic model, in which

- the primary cosmological constant with the characteristic scale of the order of grand unification energy corresponds to the flat potential of real scalar field-module, that possesses the non-trivial superpotential, whose contribution is modified after the account for the leading corrections in gravitational constant within the supergravity,
- the fine tuning of superpotential parameters should be investigated with respect of stability, and it is broken due to quantum loop-corrections, that leads to the instability of primary cosmological constant,
- the primary cosmological constant relaxes due to the inflation caused by the instability of potential for the module field playing the role of inflaton,
- after the account for quantum loop-corrections, the parameters of potential naturally agree with the values phenomenologically observed in the description of large scale structure of Universe, by the order of magnitude.

The existence of primary superpotential for the module field is determined by the relation between its parameters, which is called the cosmological bootstrap.

In the model offered, there are only two significant parameters being the scales of energies: the Planckian mass \tilde{m}_{P1} and the scale of primary cosmological constant of the order of energy in the grand unification Λ constrained by the strict hierarchy $\Lambda/\tilde{m}_{P1} \ll 1$. This proposition is enough for the realization of cosmological bootstrap as well as for the natural explanation of inflaton parameters.

In the second part of paper we have performed calculations of 1-loop structure of theory. In this way we have found the conditions allowing for the necessary hierarchy of loop corrections suitable for the cosmological bootstrap in order to break down the fine tuning of primary superpotential: the cut-off in loops with the inflaton should be about Λ , while in loops with graviton, gravitino and inflatino one should suppress the cut-off by the factor of $(\Lambda/\tilde{m}_{P1})^2$.

We have to note several technical issues: first, we have considered the corrections to the potential due to the supergravity in the leading order in the constant of gravitational interaction. Second, we have taken into account for contributions into the renormalizable terms of scalar field self-action, i.e. into the self-action below the quartic one. In this way, we have assumed that the self-action of higher powers in inflaton is beyond the control, because such the terms of lagrangian permit an arbitrary final renormalization, therefore, we have assumed them to be negligibly small in our model. Third, the scheme with the fine tuning of potential parameters, i.e. the cosmological bootstrap, includes the fixed expansion of parameters in higher degrees of small ratio of cosmological scale to the Planckian mass Λ/\tilde{m}_{P1} . Therefore, the introduction of loop corrections in Λ/\tilde{m}_{P1} breaks the structure of cosmological bootstrap and generates the conditions of inflation. In this respect, the offered scheme for the relation of primary cosmological constant to the inflation looks like reasonable, in a whole.

This work is in part supported by the RFBR grants 10-02-00061, the grant of Russian Federal Program ‘‘Science and Education personal’’ for the Center of Science and Education 2009-1.1-125-055-008; the work by S.A.T. is supported by the grant of Russian President MK-406.2010.2.

Appendix I: An imaginary part of scalar field and a general form of potential

With no restriction of generality in the consideration of superpotential (12) one can put the parameter Λ to be real, while μ_0 and g_0 should be generically set complex. Then, the potential gets the form

$$V_S = \left| \frac{\partial W}{\partial \Phi} \right|^2 = \Lambda^4 + g_0 g_0^* (\Phi^* \Phi)^2 + \mu_0 \mu_0^* \Phi^* \Phi + \Lambda^2 \{ g_0^* (\Phi^*)^2 + g_0 \Phi^2 \} + i \{ \mu_0 \Phi - \mu_0^* \Phi^* \} + i \Lambda^2 \Phi^* \Phi \{ \mu_0 g_0^* \Phi^* - \mu_0^* g_0 \Phi \}, \quad (I1)$$

so that the discrete symmetry with respect to the inversion $\Phi \leftrightarrow -\Phi^*$ leads to the following conditions:

$$\mu_0^* = \mu_0, \quad g_0^* = g_0, \quad (I2)$$

and the potential equals

$$V_S = \Lambda^4 + \frac{1}{2} \{ \mu_0^2 + 2g_0 \Lambda^2 \} \phi^2 + \frac{1}{4} g_0^2 \phi^4 + \Delta V_S, \quad (I3)$$

where the additional term is equal to

$$\Delta V_S = \mu_0 \{g_0 \phi^2 - 2\Lambda^2\} \tilde{\phi} - 4g_0 \Lambda^2 \tilde{\phi}^2, \quad (\text{I4})$$

as expressed in real fields

$$\phi = \sqrt{2}|\Phi|, \quad \tilde{\phi} = \Im \Phi, \quad (\text{I5})$$

hence, there is the constraint

$$|\tilde{\phi}| \leq \frac{1}{\sqrt{2}} \phi. \quad (\text{I6})$$

In the model under study we have got $g_0 < 0$, therefore, the term ΔV_S has got the minimum versus $\tilde{\phi}$ at a fixed absolute value of Φ . In this way, the nil value of imaginary part for the field is not stable.

The minimum versus $\tilde{\phi}$ is posed at

$$\tilde{\phi}_* = \mu_0 \frac{g_0 \phi^2 - 2\Lambda^2}{8g_0 \Lambda^2}, \quad (\text{I7})$$

hence, this field can be approximated by a constant

$$\tilde{\phi}_* \approx -\frac{\mu_0}{4g_0}, \quad (\text{I8})$$

in the region of

$$\phi^2 \ll -\frac{\Lambda^2}{g_0}, \quad (\text{I9})$$

if $\phi^2 > \mu_0^2/8g_0^2$. In the realistic model we get $g_0 \sim -(\Lambda/\tilde{m}_{\text{Pl}})^6$, and the constraint of (I9) is reduced to $\phi \ll \tilde{m}_{\text{Pl}}(\tilde{m}_{\text{Pl}}/\Lambda)^2$, i.e. to the following condition: the field is certainly less than its vacuum expectation value.

The substitution of (I7) gives the potential for the field ϕ ,

$$V_S \mapsto \frac{\lambda_0}{4} (\phi^2 - \phi_0^2)^2, \quad (\text{I10})$$

where

$$\phi_0^2 = -\frac{2\Lambda^2}{g_0}, \quad \lambda_0 = g_0 \left(g_0 + \frac{\mu_0^2}{4\Lambda^2} \right), \quad (\text{I11})$$

i.e. the potential has got zero value of vacuum energy as should be in the case of single chiral superfield, when one can find a suitable complex solution of quadratic equation $\partial W/\partial \Phi = 0$ with respect to Φ , that coincides with $\phi_0 = |\Phi|$, of course.

Thus, we have draw the conclusion that in the case of actual cosmological role for the scalar field, the phenomenological requirement on “freezing out” its imaginary part in the dynamics means the introduction of term breaking down the supersymmetry, that leads to $\tilde{\phi} \rightarrow 0$. We can reach this purpose, for instance, by adding “a massive term” in the form

$$\Delta \tilde{V} = \tilde{m}^2 \left(\tilde{\phi} + C \mu_0 \frac{\Lambda^2}{\tilde{m}^2} \right)^2 + \text{const.}, \quad (\text{I12})$$

where the mass has got a value of the order of Planck scale of energy, $\tilde{m} \sim \tilde{m}_{\text{Pl}}$. The first variant of $\Delta \tilde{V}$ with $C = 0$ does not involve any fine tuning and it gives a negligible value of imaginary part for the scalar field $\tilde{\phi}_* \sim \mu_0 \Lambda^2/\tilde{m}_{\text{Pl}}^2 \rightarrow 0$, that weakly depends on ϕ , while the second variant with $C = 1$ leads to the cancelation of term linear in $\tilde{\phi}$ in the potential independent of the real part of field, hence, one gets the stronger suppression of imaginary part. The second variant essentially extends the region of applicability for the approximation with zero value of $\tilde{\phi}$. Finally, in the third variant at $C = 2 + \mu_0^2/(2g_0\Lambda^2)$ the imaginary part becomes equal to zero in the vacuum, i.e. in the minimum of potential for the real part of field at $\phi_*^2 = -2\Lambda^2/g_0 - \mu_0^2/g_0^2 > 0$, then, in this phenomenologically actual case, the vacuum state becomes invariant with respect to the operation of complex conjugation for Φ . In this way, the dynamics

of imaginary part is absolutely inessential at the energies much less than the Planckian mass, i.e. in the classical description of gravity.

Thus, the involvement of scalar field in the cosmological model allows us to assign this field to real values, indeed.

Finally, let us note that the introduction of mass for the imaginary part of scalar field along with the breaking down the supersymmetry leads to the standard sum rules for the squares of masses for the components of chiral superfield with the fermionic number $F = \{0, 1\}$

$$\sum (-1)^F m^2 = 0 \quad \Rightarrow \quad m^2 + \tilde{m}^2 = 2m'^2, \quad (\text{II3})$$

where m' denotes the mass of scalar field superpartner being the Majorana field of inflatino. Therefore, the inflatino inevitably gets the mass of the order of Planck scale.

Appendix II: A chiral rotation

The superpartner of scalar field, i.e. the inflatino, is the Majorana spinor ψ . It is the charge self-conjugated spinor possessing the left-handed and right-handed components in the chiral representation

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} = \begin{pmatrix} \chi \\ \bar{\chi} \end{pmatrix}, \quad (\text{II1})$$

where the 2-component spinors are charge conjugated to each other, $\bar{\chi} = i\sigma_2 \chi^*$, so that the superpotential (12) leads to the terms of lagrangian quadratic in the inflatino

$$\mathcal{L}_2 = -\frac{1}{2} (i\mu_0 \chi \chi - i\mu_0 \bar{\chi} \bar{\chi} + 2g_0 \Phi \chi \chi + 2g_0 \Phi^* \bar{\chi} \bar{\chi}). \quad (\text{II2})$$

This lagrangian can be reduced to the standard form with the real mass of Majorana field, if we make the chiral rotation

$$\psi_u = e^{i\gamma_5 u} \psi \quad \Rightarrow \quad \chi_u = e^{iu} \chi, \quad \bar{\chi}_u = e^{-iu} \bar{\chi}, \quad (\text{II3})$$

with the parameter $u = -\pi/4$, so that

$$\mathcal{L}_2 = -\frac{1}{2} (\mu_0 \chi_u \chi_u + \mu_0 \bar{\chi}_u \bar{\chi}_u - 2ig_0 \Phi \chi_u \chi_u + 2ig_0 \Phi^* \bar{\chi}_u \bar{\chi}_u) \quad (\text{II4})$$

$$= -\frac{1}{2} (\mu_0 \bar{\psi}_R \psi_L + \mu_0 \bar{\psi}_L \psi_R - 2ig_0 \Phi \bar{\psi}_R \psi_L + 2ig_0 \Phi^* \bar{\psi}_L \psi_R) \quad (\text{II5})$$

$$= -\frac{1}{2} (\mu_0 \bar{\psi} \psi - \sqrt{2} i g_0 \phi \bar{\psi} \gamma_5 \psi + 2g_0 \tilde{\phi} \bar{\psi} \gamma_5 \psi), \quad (\text{II6})$$

where, for the sake of brevity of notations, we have omitted the subscript u , that is not essential for our presentation.

Thus, after the chiral rotation we have got the Majorana field with the definite vertexes of Yukawa interaction with the real and imaginary parts of scalar field in the chiral superfield. In this respect, it is important to emphasize that the kinetic terms for the left-handed and right-handed components of spinor remain invariant under the chiral rotation. Moreover, the contact terms of inflatino coupling to the gravitino and scalar field are also invariant, that is caused by the fact that the gravitino is described by the Majorana field, too, and the contact terms of interaction preserve the chirality.

Appendix III: Feynman rules

Let us describe the rules for diagrams. We use the definition of metrics and Dirac matrices by S. Weinberg [61]:

$$\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1), \quad \{\gamma_\mu, \gamma_\nu\} = 2\eta_{\mu\nu}. \quad (\text{III1})$$

The propagator of inflaton is denoted by the solid line corresponding to the expression

$$(-i) \frac{1}{p^2 + m^2}, \quad (\text{III2})$$

where m is the inflaton mass.

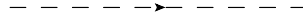
The inflatino is denoted by the double line with the propagator of the form



$$(-i) \frac{-i\gamma p + m'}{p^2 + m'^2}, \quad (\text{III3})$$

where m' stands for the inflatino mass.

The gravitino is shown by the dashed line with the propagator



$$\begin{aligned} (-i) \frac{P^{\mu\nu}(p)}{p^2 + m_g^2} = & (-i) \frac{1}{p^2 + m_g^2} \left\{ \left(\eta^{\mu\nu} + \frac{p^\mu p^\nu}{m_g^2} \right) (-i\gamma p + m_g) \right. \\ & \left. - \frac{1}{3} \left(\gamma^\mu - i \frac{p^\mu}{m_g} \right) (i\gamma p + m_g) \left(\gamma^\nu - i \frac{p^\nu}{m_g} \right) \right\}, \end{aligned} \quad (\text{III4})$$

where $m_g = \sqrt{\frac{2\pi G}{3}} \Lambda^4$ is the gravitino mass.

The vertexes with the gravitino appear by involving the supersymmetry so that the gravitino interacts with the supercurrent through the term of lagrangian

$$\sqrt{8\pi G} \int d^4x \frac{1}{2} \bar{S}^\mu \psi_\mu, \quad (\text{III5})$$

where ψ^μ denotes the gravitino field, while the supercurrent equals

$$S^\mu = \sqrt{2} \left[\gamma^\nu \partial_\nu \phi \gamma^\mu \psi_R + \gamma^\nu \partial_\nu \phi^* \gamma^\mu \psi_L + \left(\frac{\partial W}{\partial \phi} \right) \gamma^\mu \psi_L + \left(\frac{\partial W}{\partial \phi} \right)^* \gamma^\mu \psi_R \right],$$

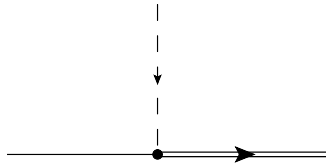
here ϕ denotes the inflaton, ψ does the inflatino, and W is the superpotential.

The lagrangian contains the potential terms corresponding to the inflaton self-action and the coupling of inflaton to the inflatino

$$\int d^4x \left\{ \frac{1}{2} \left(\frac{\partial^2 W}{\partial \Phi^2} \right) (\bar{\psi}_L \psi_L) + \frac{1}{2} \left(\frac{\partial^2 W}{\partial \Phi^2} \right)^* (\bar{\psi}_L \psi_L)^* + \left(\frac{\partial W}{\partial \Phi} \right) \left(\frac{\partial W}{\partial \Phi} \right)^* \right\}. \quad (\text{III6})$$

There are three kinds of vertexes appearing from the supercurrent:

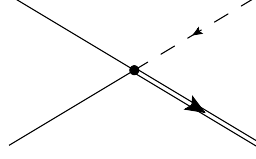
- three lines of inflaton, inflatino and gravitino come into the first vertex, so that we get



$$(i)(i) \frac{1}{2} \mu_0 \sqrt{8\pi G} \gamma^\nu \gamma^5 = -\frac{1}{2} \mu_0 \sqrt{8\pi G} \gamma^\nu \gamma^5, \quad (\text{III7})$$

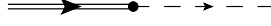
where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$;

- four lines of inflatino, gravitino and twice inflaton due to its self-action compose the second vertex



$$(i) \frac{1}{\sqrt{2}} g_0 \sqrt{8\pi G} \gamma^\nu, \quad (\text{III8})$$

- the inflatino is transformed into the gravitino in the third vertex



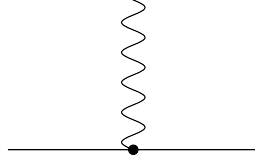
$$i \frac{1}{\sqrt{2}} \sqrt{8\pi G} \Lambda^2 \gamma^\nu. \quad (\text{III9})$$

The propagator of graviton looks like



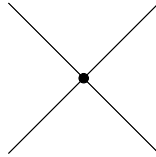
$$G^{\mu\nu, \mu'\nu'} = (-i) \frac{1}{2p^2} (\eta^{\mu\mu'} \eta^{\nu\nu'} + \eta^{\mu\nu'} \eta^{\nu\mu'} - \eta^{\mu\nu} \eta^{\mu'\nu'}). \quad (\text{III10})$$

The vertex of coupling the graviton to the scalar particle is determined by the term of lagrangian $\sqrt{8\pi G} \int d^4x T^{\mu\nu} h_{\mu\nu}$, where $T^{\mu\nu}$ is the tensor of energy-momentum, while $h_{\mu\nu}$ is given by $g_{\mu\nu} = \eta_{\mu\nu} + 2\sqrt{8\pi G} h_{\mu\nu}$, so that the vertex is equal to



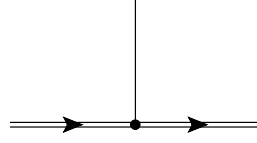
$$(-i) \sqrt{8\pi G} \eta^{\mu\nu} (\mu_0^2 + 2g_0 \Lambda^2). \quad (\text{III11})$$

The quartic self-action gives the vertex



$$-i\lambda_0 = -6ig_0^2, \quad (\text{III12})$$

and the vertex for the interaction between the inflaton and inflatino appears from the lagrangian after the chiral rotation (Appendix II):



$$- \sqrt{2} g_0 \gamma^5. \quad (\text{III13})$$

All of vertexes preserve the conservation of momentum. The indefinite momenta are integrated out as $\frac{d^4 p}{(2\pi)^4}$. Each fermionic loop makes the factor (-1) . The Wick rotation corresponds to the substitution of $p_0 \rightarrow ip_4$ in the transformation to the Euclidean coordinates.

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